A New Analytical Model for Multifiber WDM Networks

Ling Li, *Student Member, IEEE* and Arun K. Somani, *Fellow, IEEE*

***Abstract*—We study the effect of multiple fibers in circuit-switched all-optical wavelength-routing networks. A new analytical model—the multifiber link-load correlation (MLLC) model—is developed to evaluate the blocking performance of such networks. To our knowledge, the MLLC model is the first model that takes the link-load correlation into account in multifiber WDM networks. We show that the MLLC model is accurate for a variety of network topologies by comparing the analytical results to simulation results. We observed that a small number of fibers are sufficient to guarantee high network performance in multifiber WDM networks.**

***Index Terms*—Call blocking performance, link-load correlation, multifiber networks, optical networking.**

I. INTRODUCTION

ALL-OPTICAL networks employing wavelength-division multiplexing and wavelength routing are a viable solution for future wide-area networks (WANs) and metropolitan-area networks (MANs). In WDM networks, each node has a dynamically reconfigurable photonic switch. Signals are kept in the optical domain from a source node to a destination node, thereby providing end-to-end protocol and data format transparency, and simplified management and processing compared to routing in systems using digital cross-connects [1].

In wavelength-routed all-optical WDM networks, a *lightpath* is an "optical communication path" between two nodes, established by allocating the same wavelength throughout the route of the transmitted data [2]. The requirement that the same wavelength must be used on all the links along the selected path is known as the wavelength continuity constraint. Two lightpaths can share the same fiber link, only if they use different wavelengths. A connection request encounters high performance degradation because of the wavelength continuity constraint. If each intermediate node on a path is equipped with a wavelength converter, the wavelength continuity is not required. However, the technology of all-optical wavelength conversion is not mature yet. The cost of an all-optical wavelength converter is likely to remain high in the near future. Sparse wavelength conversion and limited wavelength conversion are proposed to reduce the cost of wavelength converters in [3] and [4], respectively. Multifiber WDM networks are an alternative solution to overcome the wavelength continuity constraint. In multifiber WDM networks, each link consists of multiple fibers, and each fiber carries information on multiple wavelengths. A wavelength that cannot continue on the next hop can be switched to another fiber using an optical cross-connect (OXC) if the same wavelength is free on one of the other fibers.

Much research has been done in obtaining the call blocking performance of single-fiber WDM networks [3], [5]–[10]. A Markov chain (MC) model with the consideration of link-load correlation in [3] is accurate, and has a moderate complexity. As pointed out in [8], the MC model is an approximate model, because the arrival rates vary with the state of the Markov chain. However, the approximation does not affect the accuracy significantly. Analytical models have also been proposed in [4], [11]–[14] to study the performance of WDM networks with limited wavelength conversion. A simple analytical model is developed in [4] for two-hop and multihop paths. A model to compute blocking probabilities for mesh-torus networks is presented in [13]. A more general model that can be used in any topology is proposed in [14]. However, this model can only be used in networks with small values of conversion degree, because of the approximation made in the model. Since link-load correlation is not considered, the analysis may not be accurate for sparse network topologies. There has also been considerable interest in analyzing the blocking performance of multifiber WDM networks. The independent wavelength load model in [6] is extended to multifiber networks in [15]. The results of this model are not numerically accurate for Poisson traffic because of the assumption that the load on one wavelength is independent of those on the other wavelengths on a link. The link load independence model proposed in [5] is extended to multifiber networks in [16]. However, this independent model is not accurate [16]. It overestimates the blocking performance for $F = 1$ and underestimates for $F > 1$ in a mesh-torus network. The blocking performance models for first-fit wavelength assignment in [17] and [18] are also proposed to be applicable in multifiber networks. However, both of these models require intensive computation due to their iterative procedure to solve the Erlang fixed-point equation. Multifiber WDM networks have also been studied in [19]–[21].

We study the effect of multiple fibers in circuit-switched all-optical WDM networks. Our study method is different from the ones used in [15] and [16], which assume that both F and W are fixed for a network. We assume that the traffic load of a network is fixed, and the number of channels to support this traffic, $C = FW$, is a constant and known beforehand. We vary F from 1 (no wavelength conversion), etc., to FW (full-range wavelength conversion), and $W = C/F$ accordingly. Our study method shows clearly the effect of multiple fibers on the network per-

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formance. A network operator could easily pick up an F and W combination with the consideration of both cost and network performance. A new analytical model—a multifiber link-load correlation (MLLC) model—is proposed to compute the blocking performance of multifiber networks in this paper. The MLLC model is based on the Markov chain (MC) model for single-fiber networks proposed in [3]. Similar to the Markov chain model, we start by considering a two-hop path. We use hop and link interchangeably throughout this paper. Since we assume that the load on the lth hop is dependent only on the load on the $(l-1)$th hop, the probability of blocking on the l-hop path can be computed using the results for a two-hop path. This is done by viewing the first $(l-1)$ hops as the first hop and the lth hop as the second hop of a two-hop path. The difficulty in the study of a multifiber two-hop path results from the continuing calls from the first hop to the second hop. To simplify the study, we divide the wavelengths on the two-hop path into different groups. The distribution of continuing calls is computed in each group.

To our knowledge, this is the first analytical model taking the link-load correlation into account in multifiber WDM networks. Comparing to the link independence model in [16] and [14], our model is an accurate and general model that is applicable not only to regular networks but also irregular networks.

This paper is organized as follows. A multifiber link-load correlation model is developed to analyze the performance of multifiber WDM networks in Section II. The model is applied to a variety of network topologies and the results are shown in Section III. We observed that the analytical results obtained from the MLLC model match closely with simulation results for the typical network topologies we studied. The accuracy of the analytical results is significantly improved from the MLLC compared to the link independence model proposed in [16]. We also observed that limited number of fibers is sufficient to guarantee high network performance in multifiber WDM networks. We present our final remarks in Section IV.

II. A Multifiber Link-Load Correlation Model

We propose a new analytical model to compute the blocking performance of multifiber WDM networks in this section. In the analytical model, we assume Poisson input traffic with arrival rate λ at each node and exponentially distributed call holding time with mean $1/\mu$. A single path is preselected for each source–destination (s–d) pair, and a wavelength assigned to a connection is uniformly randomly selected from the set of free wavelengths on that path. Let F be the number of fibers per link, and W be the number of wavelengths per fiber. We assume that F and W are the same on all links and fibers of a network. We also assume that an incoming request on one input port can be switched to any output port using OXC as long as the output port has the same wavelength free regardless of which fiber it is on. If the wavelength is not free on all of the F fibers at the output, the request is blocked on this wavelength. No wavelength converter is available at any node.

We define a light channel (LC) as a wavelength on a fiber on a link. A lightpath (LP), defined in previous section, is a connection between a source–destination (s–d) pair using the same wavelength on all the links of a path. Note that a lightpath

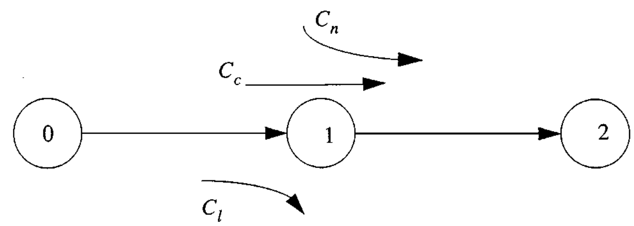

Fig. 1. Calls arriving and leaving on a two-hop path.

consists of several LCs on successive links. The LCs on a path may or may not be on the same fiber. Let a wavelength trunk (WT) λ_i be a collection of the LCs/LPs using wavelength λ_i on all the fibers. Fig. 1 shows a two-hop path with W WTs on each link, i.e., each WT consists of F fibers. We define a WT "free" on a *link* if the wavelength is free on at least one of the fibers on the link. A WT is "busy" on the link otherwise. A WT is "free" on a *path* if that WT is free on all of the links constituting the path. A WT is "busy" on the path otherwise.

In the analytical model, we start by analyzing a two-hop path as shown in Fig. 1. The states of a two-hop path can be modeled using a three-dimensional Markov chain as follows. Let C_l be the number of calls that enter the path at node 0 and leave at node 1. Let C_c be the number of calls that enter the path at node 0 and continue on to the second link. And let C_n be the number of calls that enter the path at node 1. Therefore, the number of calls that use the first link is $C_l + C_c$ and the number of calls that use the second link is $C_c + C_n$. Since the number of calls on a link cannot exceed the total number of available light channels, FW, we have $C_l + C_c \le FW$, and $C_c + C_n \le FW$.

The following conditional probabilities are defined for the three-dimensional MC on a two-hop path.

- $R_{WTLC}(\hat{N}_{f_2} | \hat{X}_{f_1}, z_{c_2}, y_{f_2})$ = Pr{the probability that $\hat{N}_{f_2}$ WTs are free on a two-hop path $|\hat{X}_{f_1}$ WTs are free on the first hop of the path, y_{f_2} LCs are free on the second hop, and z_{c_2} LCs are busy on both of the links occupied by continuing calls from the first to the second hop}.[1]
- $U(z_{c_2} | x_{f_1}, y_{f_2})$ = Pr{z_{c_2} LCs are occupied by continuing calls from the first link to the second link $|x_{f_1}$ LCs are free on the first link, and y_{f_2} LCs are free on the second link}, see (1) at the bottom of the next page.
- $S(y_{f_2} | x_{f_1})$ = Pr{y_{f_2} LCs are free on the second link $|x_{f_1}$ LCs are free on the first link of the path}, see (2) at the bottom of the next page.

Here $U(z_{c_2} | x_{f_1}, y_{f_2})$ and $S(y_{f_2} | x_{f_1})$ are functions of the steady- state probability of state $\pi(c_l, c_c, c_n)$ that is given by [22]

$$\pi(c_l, c_c, c_n) = \frac{\frac{\left(\frac{\lambda_l}{\mu}\right)^{c_l}}{c_l!} \frac{\left(\frac{\lambda_c}{\mu}\right)^{c_c}}{c_c!} \frac{\left(\frac{\lambda_n}{\mu}\right)^{c_n}}{c_n!}}{\sum_{j=0}^{WF} \sum_{i=0}^{WF-j} \sum_{k=0}^{WF-j} \frac{\left(\frac{\lambda_l}{\mu}\right)^{i}}{i!} \frac{\left(\frac{\lambda_c}{\mu}\right)^{j}}{j!} \frac{\left(\frac{\lambda_n}{\mu}\right)^{k}}{k!}}$$
$$0 \le c_l + c_c \le WF, \; 0 \le c_c + c_n \le WF \quad (3)$$

[1]In this paper, we put a hat on the variables for the number of WTs to differentiate them from the variables for the number of LCs on a link.

where λ_l, λ_c, λ_n are the arrival rates of calls that leave the first link, continue from the first link to the second link, and enter at the second link, respectively. $1/\mu$ is the expected value of the exponentially distributed call holding time.

Because of the assumption that the load on the lth hop is dependent only on the load on the $(l-1)$th hop, the blocking probability on an l-hop path can be computed recursively by viewing the first $l-1$ hops as the first hop and the lth hop as the second hop of a two-hop path. Then we focus on the analysis of a two-hop path, i.e., the computation of $R_{WTLC}(\hat{N}_{f_2}|\hat{X}_{f_1}, z_{c_2}, y_{f_2})$ in the following. The following notations are needed to obtain the blocking probabilities.

- Let w be the number of considered WTs on a link, $w \le W$. w is used as a subscript in the expressions of this paper to indicate that the computation of the expressions is on w WTs.
- Let $z_{c_2}^p$ be the number of continuing calls on a two-hop path which occupy only part of a WT, i.e., do not use all the F LCs on any WT. In other words, no busy WT is occupied completely by $z_{c_2}^p$ calls.
- Let y_{b_2} be the number of busy LCs occupied by entering calls on the second link of the two-hop path. We know that $y_{b_2} = WF - y_{f_2} - z_{c_2}$.

A. Computation of $R_{WTLC}(\hat{N}_{f_2}|\hat{X}_{f_1}, z_{c_2}, y_{f_2})$

The difficulty in computing $R_{WTLC}(\hat{N}_{f_2}|\hat{X}_{f_1}, z_{c_2}, y_{f_2})$ results from the continuing calls from the first hop to the second hop. To simplify the computation, we divide the W WTs on the two-hop path into different groups. The conditional distribution of continuing calls is computed in each group.

We first divide the W WTs on the two-hop path into two groups as shown in Fig. 2. The first group, $G(\hat{Z}_{c_2})$, consists of $\hat{Z}_{c_2}$ busy WTs that are occupied completely by continuing calls on both of the links. The other $(W - \hat{Z}_{c_2})$ WTs belongs to the

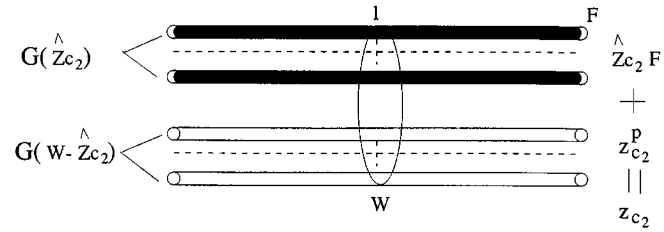

Fig. 2. The W WTs are divided into two groups.

second group, $G(W - \hat{Z}_{c_2})$, which has $(z_{c_2} - \hat{Z}_{c_2}F)$ continuing calls but no busy WT is occupied completely by the continuing calls. $R_{WTLC}(\hat{N}_{f_2}|\hat{X}_{f_1}, z_{c_2}, y_{f_2})$ is given by

$$R_{WTLC}(\hat{N}_{f_2}|\hat{X}_{f_1}, z_{c_2}, y_{f_2}) = \sum_{\hat{Z}_{c_2}=0}^{\lfloor z_{c_2}/F \rfloor} P_1(\hat{Z}_{c_2}|\hat{X}_{f_1}, z_{c_2}, y_{f_2})_W \cdot P_2(\hat{N}_{f_2}|\hat{X}_{f_1}, z_{c_2}^p, y_{f_2})_{W-\hat{Z}_{c_2}} \quad (4)$$

where $z_{c_2}^p = z_{c_2} - \hat{Z}_{c_2}F$. Here $P_1(\hat{Z}_{c_2}|\hat{X}_{f_1}, z_{c_2}, y_{f_2})_{w=W}$ is the probability that $\hat{Z}_{c_2}$ busy WTs are occupied completely by continuing calls given that $\hat{X}_{f_1}$ WTs are free on the first hop of the path, y_{f_2} LCs are free on the second hop, and z_{c_2} busy LCs continue from the first to the second hop. We assume here that the calls are arranged on the two links such that all possible LC configurations to reach the state $(\hat{X}_{f_1}, z_{c_2}, y_{f_2})$ are equiprobable.[2] Thus we can distribute the z_{c_2} continuing calls to the two links first, then distribute the leaving and entering calls on the

[2]The arrival rates actually change with the state of the Markov chain because of blocking on other links of the network. However, modeling it in multifiber case would considerably complicate the model. We show later that the loss in accuracy due to our assumption is not significant.

$$U(z_{c_2}|x_{f_1}, y_{f_2}) = \Pr(C_c = z_{c_2}|C_l + C_c = WF - x_{f_1}, C_n + C_c = WF - y_{f_2}) = \frac{\pi(WF - x_{f_1} - z_{c_2}, z_{c_2}, WF - y_{f_2} - z_{c_2})}{\sum_{x_{c_2}=0}^{\min(WF-x_{f_1}, WF-y_{f_2})} \pi(WF - x_{f_1} - x_{c_2}, x_{c_2}, WF - y_{f_2} - x_{c_2})} \quad (1)$$

$$S(y_{f_2}|x_{f_1}) = \Pr(C_n + C_c = WF - y_{f_2} \mid C_l + C_c = WF - x_{f_1}) = \frac{\sum_{x_{c_2}=0}^{\min(WF-x_{f_1}, WF-y_{f_2})} \pi(WF - x_{f_1} - x_{c_2}, x_{c_2}, WF - y_{f_2} - x_{c_2})}{\sum_{x_{c2}=0}^{WF-x_{f_1}} \sum_{x_{n_2}=0}^{WF-x_{c_2}} \pi(WF - x_{f_1} - x_{c_2}, x_{c_2}, x_{n_2})} \quad (2)$$

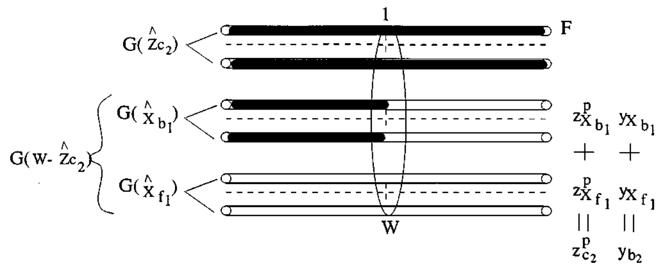

Fig. 3. The WTs in the $G(W-\hat{Z}_{c_2})$ group are divided further into two groups.

first and second link to reach the state $(\hat{X}_{f_1}, z_{c_2}, y_{f_2})$. $P_1(\hat{Z}_{c_2}|\hat{X}_{f_1}, z_{c_2}, y_{f_2})_{w=W}$ can be computed as

$$P_1(\hat{Z}_{c_2}|\hat{X}_{f_1}, z_{c_2}, y_{f_2})_w = \begin{cases} 0 & \hat{Z}_{c_2} + \hat{X}_{f_1} > W \\ 0 & y_{f_2} + z_{c_2} > WF \\ \dfrac{\dbinom{w}{\hat{Z}_{c_2}} f(z_{c_2}^p, w-\hat{Z}_{c_2}, F)}{\dbinom{wF}{z_{c_2}}} & \text{otherwise} \end{cases} \quad (5)$$

where $z_{c_2}^p = z_{c_2} - \hat{Z}_{c_2}F$, and $f(z,k,F)$ is the number of ways of distributing z LCs to k WTs such that no WT is fully occupied by the continuing calls. Note that each WT consists of F fibers. Suppose $j = \lfloor z/F \rfloor$. $f(z,k,F)$ is given by [16]

$$f(z,k,F) = \begin{cases} 0 & z > (F-1)k \\ \dbinom{kF}{z} & z < F \\ \dbinom{kF}{z} - \displaystyle\sum_{i=1}^{j} \dbinom{k}{i} f(z-iF, k-i, F) & \text{otherwise.} \end{cases} \quad (6)$$

In (4), $P_2(\hat{N}_{f_2}|\hat{X}_{f_1}, z_{c_2}^p, y_{f_2})_{W-\hat{Z}_{c_2}}$ is the probability that $\hat{N}_{f_2}$ WTs are free on a two-hop path with $w = W - \hat{Z}_{c_2}$ WTs, given that $\hat{X}_{f_1}$ WTs are free on the first hop of the path, y_{f_2} LCs are free on the second hop, $z_{c_2}^p$ calls continue from the first to the second hop but no busy WT is occupied completely by these continuing calls. To compute $P_2(\hat{N}_{f_2}|\hat{X}_{f_1}, z_{c_2}^p, y_{f_2})_{W-\hat{Z}_{c_2}}$, we divide the second WT group further into $G(\hat{X}_{b_1})$ group and $G(\hat{X}_{f_1})$ group as shown in Fig. 3. $G(\hat{X}_{b_1})$ group consists of $\hat{X}_{b_1} = W - \hat{X}_{f_1} - \hat{Z}_{c_2}$ busy WTs on the first hop, occupied by continuing calls, or leaving calls, but no busy WT is occupied completely by continuing calls. The WTs in the $G(\hat{X}_{f_1})$ group are free WTs on the first hop. The WTs in these two groups may or may not busy on the second hop. Recalling our definition of a "free" WT, some LCs may be set up on the WTs in $G(\hat{X}_{f_1})$, but the number of busy LCs on a WT is less than F. Let $z_{X_{b_1}}^p$ be the number of continuing calls distributed in the $G(\hat{X}_{b_1})$ group. Then the number of continuing calls distributed on the $G(\hat{X}_{f_1})$ group is $z_{X_{f_1}}^p = z_{c_2}^p - z_{X_{b_1}}^p$. Recall that y_{b_2} is the busy LCs occupied by entering calls on the second link of the two-hop path. Let $y_{X_{b_1}}$ be the number of entering calls at the second link in the $G(\hat{X}_{b_1})$ group, then the number of entering calls in the $G(\hat{X}_{f_1})$ group is $y_{X_{f_1}} = y_{b_2} - y_{X_{b_1}}$. The probability $P_2(\hat{N}_{f_2}|\hat{X}_{f_1}, z_{c_2}^p, y_{f_2})_{w=W-\hat{Z}_{c_2}}$ in (4) is given by

$$\begin{aligned} P_2(\hat{N}_{f_2}|\hat{X}_{f_1}, z_{c_2}^p, y_{f_2})_{w=W-\hat{Z}_{c_2}} &= \sum_{z_{X_{b_1}}^p=0}^{z_{c_2}^p} \sum_{y_{X_{b_1}}=0}^{wF-z_{c_2}-y_{f_2}} \\ &\cdot P_4(y_{X_{b_1}}|z_{X_{b_1}}^p, z_{X_{f_1}}^p, y_{b_2}, \hat{X}_{b_1}, \hat{X}_{f_1})_w \\ &\cdot P_3(z_{X_{b_1}}^p|z_{c_2}^p, \hat{X}_{b_1}, \hat{X}_{f_1})_w \\ &\cdot P_5(\hat{N}_{f_2}|\hat{X}_{f_1}, z_{X_{f_1}}^p, y_{X_{f_1}})_{\hat{X}_{f_1}} \end{aligned} \quad (7)$$

where

$$\hat{X}_{b_1} = W - \hat{X}_{f_1} - \hat{Z}_{c_2}, \quad z_{X_{f_1}}^p = z_{c_2}^p - z_{X_{b_1}}^p$$

and

$$y_{X_{f_1}} = y_{b_2} - y_{X_{b_1}}.$$

Here $P_3(z_{X_{b_1}}^p|z_{c_2}^p, \hat{X}_{b_1}, \hat{X}_{f_1})_w$ is the probability that $z_{X_{b_1}}^p$ continuing calls are in the subgroup $G(\hat{X}_{b_1})$ given that $z_{c_2}^p$ calls are randomly distributed in the group $G(W-\hat{Z}_{c_2})$ and no busy WT is occupied completely by $z_{c_2}^p$ calls. $P_3(z_{X_{b_1}}^p|z_{c_2}^p, \hat{X}_{b_1}, \hat{X}_{f_1})_w$ is given by

$$P_3(z_{X_{b_1}}^p|z_{c_2}^p, \hat{X}_{b_1}, \hat{X}_{f_1})_w = \frac{f(z_{X_{b_1}}^p, \hat{X}_{b_1}, F) f(z_{X_{f_1}}^p, z_{c_2}^p - \hat{X}_{b_1}, F)}{f(z_{c_2}^p, w, F)}. \quad (8)$$

$P_4(y_{X_{b_1}}|z_{X_{b_1}}^p, z_{X_{f_1}}^p, y_{b_2}, \hat{X}_{b_1}, \hat{X}_{f_1})_w$ in (7) is the probability that $y_{X_{b_1}}$ LCs are distributed in the subgroup $G(\hat{X}_{b_1})$, given $z_{X_{b_1}}^p$, $z_{X_{f_1}}^p$, and y_{b_2} calls are distributed in group $G(\hat{X}_{b_1})$, $G(\hat{X}_{f_1})$, and $G(W-\hat{Z}_{c_2})$, respectively. Thus,

$$P_4(y_{X_{b_1}}|z_{X_{b_1}}^p, z_{X_{f_1}}^p, y_{b_2}, \hat{X}_{b_1}, \hat{X}_{f_1})_w = \frac{\dbinom{\hat{X}_{b_1}F - z_{X_{b_1}}^p}{y_{X_{b_1}}} \dbinom{\hat{X}_{f_1}F - z_{X_{f_1}}^p}{y_{b_2} - y_{X_{b_1}}}}{\dbinom{wF - z_{X_{b_1}}^p - z_{X_{f_1}}^p}{y_{b_2}}}.$$

$P_5(\hat{N}_{f_2}|\hat{X}_{f_1}, z_{X_{f_1}}^p, y_{X_{f_1}})_{w=\hat{X}_{f_1}}$ in (7) is the probability that $\hat{N}_{f_2}$ WTs are free on the two-hop path with w WTs, given that all of the WTs are free on the first hop, $y_{X_{f_1}}$ LCs are free on the second hop, and $z_{X_{f_1}}^p$ calls continue from the first hop to the second hop. To compute $P_5(\hat{N}_{f_2}|\hat{X}_{f_1}, z_{X_{f_1}}^p, y_{X_{f_1}})_{w=\hat{X}_{f_1}}$, we divide the $G(\hat{X}_{f_1})$ group again into two groups. The group $G(\hat{N}_{f_2})$ consists of $\hat{N}_{f_2}$ WTs that are free on both of the first hop and the second hop. The other group, $G(\hat{N}_{b_2})$, consists of $\hat{N}_{b_2} = \hat{X}_{f_1} - \hat{N}_{f_2}$ WTs, which are free on the first hop but busy on the second hop as shown in Fig. 4. Let $z_{N_{b_2}}^p$ be the number of continuing calls distributed in the group $G(\hat{N}_{b_2})$. Then the number of continuing calls distributed in the group $G(\hat{N}_{f_2})$ is $z_{N_{f_2}}^p = z_{X_{f_1}}^p - z_{N_{b_2}}^p$. Let $y_{N_{b_2}}$ be the number of entering calls distributed on the $\hat{N}_{b_2}$ WTs. The number of en-

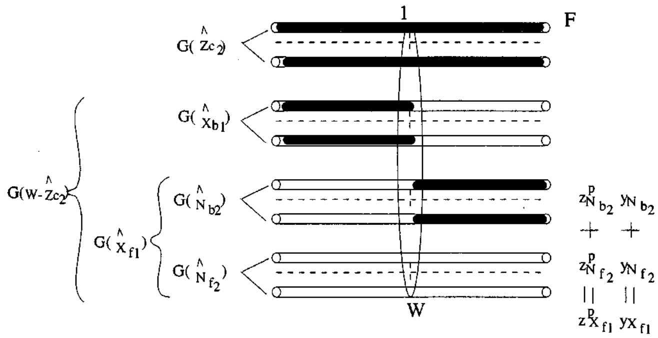

Fig. 4. The WTs in the $G(\hat{X}_{f_1})$ group are divided again into two groups.

tering calls distributed on the $\hat{N}_{f_2}$ WTs is $y_{N_{f_2}} = y_{X_{f_1}} - y_{N_{b_2}}$. $P_5(\hat{N}_{f_2}|\hat{X}_{f_1}, z^p_{X_{f_1}}, y_{X_{f_1}})_w$ is derived as

$$P_5(\hat{N}_{f_2}|\hat{X}_{f_1}, z^p_{X_{f_1}}, y_{X_{f_1}})_w = \sum_{z^p_{N_{b_2}}=0}^{z^p_{X_{f_1}}} \cdot \frac{\binom{\hat{X}_{f_1}}{\hat{N}_{f_2}} f(z^p_{N_{b_2}}, \hat{N}_{b_2}, F) g(\hat{N}_{f_2}, z^p_{X_{f_1}} - z^p_{N_{b_2}}, y_{X_{f_1}} - y_{N_{b_2}})}{f(z^p_{X_{f_1}}, \hat{X}_{f_1}, F) \binom{\hat{X}_{f_1}F - z^p_{X_{f_1}}}{y_{X_{f_1}}}}$$

where $g(\hat{N}_{f_2}, z^p_{N_{f_2}}, y_{N_{f_2}})$ is the number of ways to distribute $z^p_{N_{f_2}}$ continuing calls and $y_{N_{f_2}}$ entering calls to $\hat{N}_{f_2}$ WTs such that every WT is free. $g(\hat{N}_{f_2}, z^p_{N_{f_2}}, y_{N_{f_2}})$ is given by

$$g(\hat{N}, z, y) = \binom{\hat{N}F}{z}\binom{\hat{N}F - z}{y} - \sum_{i=1}^{\lfloor z+y/F \rfloor} \sum_{j=0}^{\min(iF, z)} \times \binom{\hat{N}}{i}\binom{iF}{j} g(\hat{N} - i, z - j, y - (iF - j)).$$

A closed-form expression of $R_{WTLC}(\hat{N}_{f_2}|\hat{X}_{f_1}, y_{f_2}, z_{c_2})$ is obtained as

$$\begin{aligned} R_{WTLC}(\hat{N}_{f_2}|\hat{X}_{f_1}, y_{f_2}, z_{c_2}) &= \sum_{\hat{Z}_{c_2}=0}^{\lfloor z_{c_2}/F \rfloor} \sum_{z^p_{X_{b_1}}=0}^{z_{c_2} - \hat{Z}_{c_2}F} \sum_{y_{X_{b_1}}=0}^{(W - \hat{Z}_{c_2})F - z_{c_2} - y_{f_2}} \\ &\cdot P_1(\hat{Z}_{c_2}|\hat{X}_{f_1}, z_{c_2}, y_{f_2}) \\ &\cdot P_3(z^p_{X_{b_1}}|z^p_{c_2}, \hat{X}_{b_1}, \hat{X}_{f_1}) \\ &\cdot P_4(y_{X_{b_1}}|z^p_{X_{b_1}}, z^p_{X_{f_1}}, y_{b_2}, \hat{X}_{b_1}, \hat{X}_{f_1}) \\ &\cdot P_5(\hat{N}_{f_2}|\hat{X}_{f_1}, z^p_{X_{f_1}}, y_{X_{f_1}}). \end{aligned} \tag{9}$$

B. Computation of Average Blocking Probability

Let $P_6^{(l)}(\hat{N}_{f_l}, y_{f_l})$ be the probability that $\hat{N}_{f_l}$ wavelengths are free on an l-hop path and y_{f_l} LCs are free on hop l. We continue to assume that the load on the lth hop is dependant only on the load on the $(l-1)$-hop. By viewing the first $l-1$ hops as the first hop and the lth hop as the second hop of a two-hop path, we can compute the blocking probability on the l-hop path using the results for a two-hop path in (9).

$$\begin{aligned} P_6^{(l)}(\hat{N}_{f_l}, y_{f_l}) = &\sum_{x_{f_{l-1}}=0}^{FW} \sum_{z_{c_l}=0}^{\min(FW - x_{f_{l-1}}, FW - y_{f_l})} \\ &\cdot \sum_{\hat{N}_{f_{l-1}}=0}^{\lfloor x_{f_{l-1}}/F \rfloor} R_{WTLC}(\hat{N}_{f_l}|\hat{N}_{f_{l-1}}, z_{c_l}, y_{f_l}) \\ &\cdot U(z_{c_l}|y_{f_l}, x_{f_{l-1}}) S(y_{f_l}|x_{f_{l-1}}) \\ &\cdot P_6^{(l-1)}(\hat{N}_{f_{l-1}}, x_{f_{l-1}}) \qquad l > 1 \end{aligned} \tag{10}$$

where $U(z_{c_2}|x_{f_1}, y_{f_2})$ and $S(y_{f_2}|x_{f_1})$ are the conditional probabilities that defined in (1) and (2), respectively. The probability that $\hat{N}_{f_2}$ wavelengths and y_{f_1} LCs are free on the first link of a path is

$$P_6^{(1)}(\hat{N}_{f_1}, y_{f_1}) = P_1(\hat{N}_{f_1}|0, 0, FW - y_{f_1})_W.$$

C. Computation of the Parameters

Let P_j^{sd} be a route between a s-d pair using link j, $P_{i,j}^{sd}$ as a route continuing from link i to link j. Let R_i be a set of fixed routes for calls that use link i, and $R_{i,j}^c$ be the set of fixed routes for calls that continue from link i to link j. λ_i^n, λ_j^l, and $\lambda_{i,j}^c$ are the arrival rates of calls entering at link i, leaving link j, and continuing from link i to link j, respectively. λ_i^n, λ_j^l, and λ_{ij}^c are given by

$$\lambda_{i,j}^c = \sum_{P_{i,j}^{sd} \in R_{i,j}^c} \lambda_{sd} \tag{11}$$

$$\lambda_i^l = \sum_{P_i^{sd} \in R_i} \lambda_{sd} - \lambda_{i,j}^c \tag{12}$$

$$\lambda_j^n = \sum_{P_j^{sd} \in R_j} \lambda_{sd} - \lambda_{i,j}^c. \tag{13}$$

These arrival rates are used to compute the steady-state probability $\pi(c_l, c_c, c_n)$ in (3). Let $l(sd)$ be the length of a path between an s–d pair. Let R be the number of s–d pairs in the network, and P_B be the networkwide average blocking probability. P_B is given by

$$P_B = \sum_{sd} \sum_{y_f=0}^{FW} P_6^{l(sd)}(0, y_f)/R. \tag{14}$$

D. Implementation and Complexity Analysis

The above equations shows an approach on how to compute the steady-state probability of a path that has i free wavelength trunks. Comparing to the link-load correlation model for single fiber networks [3], the MLLC model has the same computational complexity except for the computation of the free WT distribution on a two-hop path, $R_{WTLC}(\hat{N}_{f_2}|\hat{X}_{f_1}, y_{f_2}, z_{c_2})$. However, $R_{WTLC}(\hat{N}_{f_2}|\hat{X}_{f_1}, y_{f_2}, z_{c_2})$ does not depend on any network topology and traffic arrival rate. The only parameters needed to compute $R_{WTLC}(\hat{N}_{f_2}|\hat{X}_{f_1}, y_{f_2}, z_{c_2})$ is the number of fibers per link, F, and the number of wavelengths per fiber, W. Thus $R_{WTLC}(\hat{N}_{f_2}|\hat{X}_{f_1}, y_{f_2}, z_{c_2})$ can be computed independently. The results can be used repeatedly in different topologies and traffic patterns, as long as they have the same number of fibers per link and wavelengths per fiber.

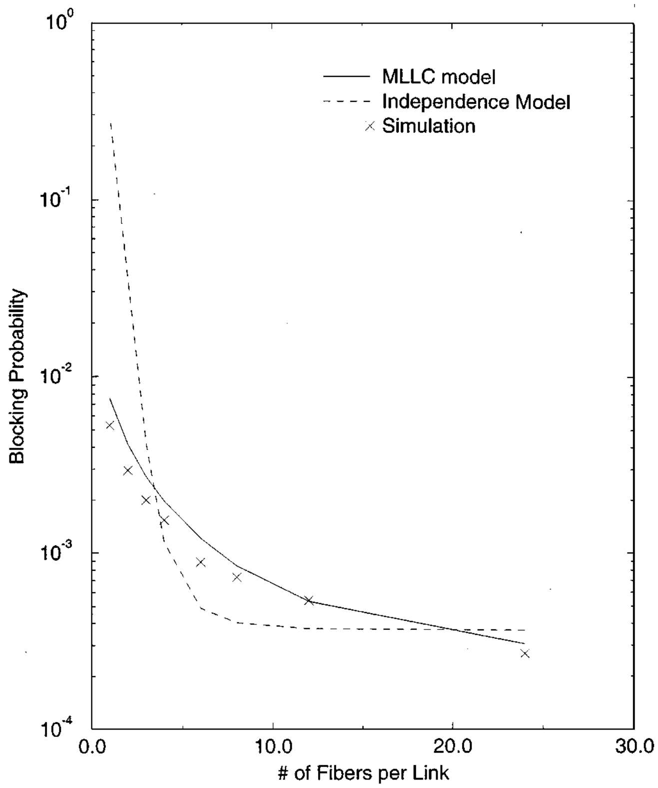

Fig. 5. Blocking probability versus number of fibers in a 10-node unidirectional ring network. The link capacity is fixed at 24.

We present a new analytical model that can be used for both regular and irregular multifiber WDM networks in this section. In the next section, we apply the analytical model to study the call blocking performance for different topologies.

III. Numerical Results and Analysis

In this section, we assess the accuracy of the multifiber link load correlation (MLLC) model by comparing it to the simulation results. We also compare the MLLC model to the multifiber independence model presented in [16], and show that the MLLC model yields more accurate results than the independence model. The MLLC model is applied to two regular topologies: the ring and the mesh-torus networks, and an irregular NSF T1 backbone network (NSFnet). We are interested in finding the effect of multifibers on these networks. The question we attempted to answer is how many fibers are required in these multifiber nonwavelength-convertible networks to provide similar performance as that of full-wavelength-convertible networks.

In the networks we studied, the link capacity is fixed at 24 light channels, i.e., $FW = 24$ on each link. We vary the number of fibers on each link, F, from 1, 2, 3, 4, 6, 8, 12 to 24, and the number of wavelengths on each fiber is varied by $W = 24/F$ accordingly. We assume Poisson traffic arrives at each node, and the destination for an arrival request is uniformly distributed among other nodes.[3] We adjust the traffic load such that the blocking probabilities are around 10^{-3}. Each data point in the simulations was obtained using 10^6 call arrivals.

[3]The MLLC model could also be used for nonuniformly distributed traffic using (11)–(13). The uniform distribution assumption is made only for simplicity. Note that *link* loads in the NSFnet are nonuniformly distributed.

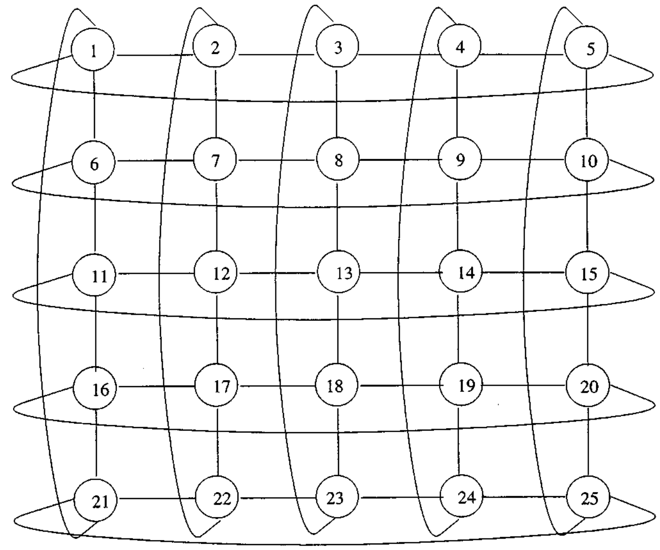

Fig. 6. A 5 × 5 mesh-torus network.

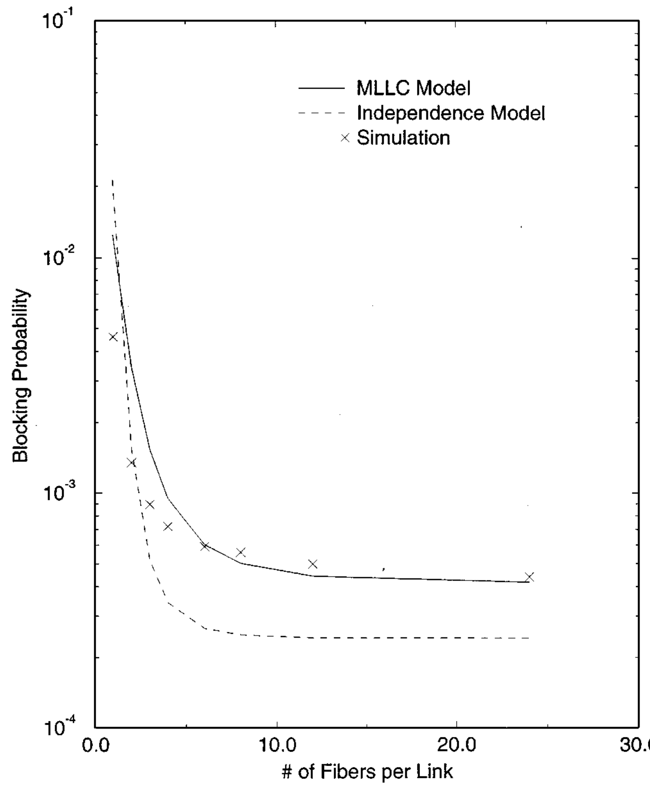

Fig. 7. Blocking probability versus number of fibers in a 5 × 5 mesh-torus network. The link capacity is fixed at 24.

We first study a 10-node unidirectional ring network in which the gain of using wavelength converters is limited. The call blocking probability against the number of fibers per link is plotted in Fig. 5. The traffic load is 2 Erlangs per node. Analytical and simulation results are plotted for different fiber–wavelength pair configurations. The close match between the analytical and simulation results and the fact that the analytical results follow the trend of the simulation results indicate that the model is adequate in analytically predicting the performance in the ring networks. For comparison, we also plot in the figure

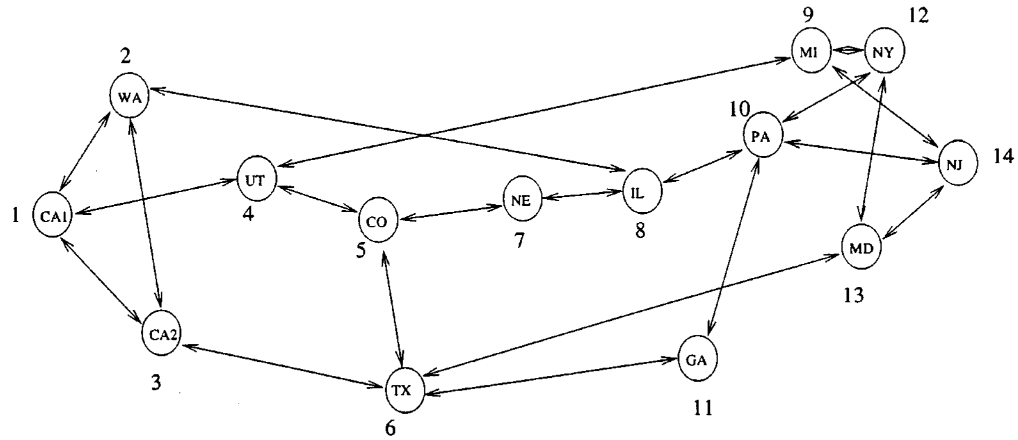

Fig. 8. The NSF T1 backbone network topology.

the analytical results obtained from the independence model in [16]. It is seen that the independence model severely overestimated the blocking probability when the number of fibers per link, F, is small, and underestimated the blocking when F becomes larger. We observed that 6 fibers per link are sufficient to provide similar performance as that in full-wavelength-convertible networks ($F = 24$, $W = 1$).

Wavelength converters are useful to reduce the blocking probability in the mesh-torus networks. A 5×5 mesh-torus network shown in Fig. 6 with node traffic of 17 Erlangs is studied. The results are shown in Fig. 7. We observed that the MLLC model performs better than the independence model, although the difference is not as much as in the ring networks. The reason is that the traffic in mesh-torus networks tends to mix well such that the effect of the link-load correlation is not significant, especially in multifiber networks. We also observed that 4 fibers are sufficient to guarantee high network performance. Recalling that multiple fibers have similar effect as limited wavelength conversion in WDM networks, our observation is coincident with the results in [4], which showed that with a conversion bandwidth that covers only 25% of the whole transmission bandwidth, the blocking probability is almost identical to the one with full-range conversion.

The above two regular topologies have been intensively studied in the literature [4], [15]. However, few analytical models are applicable in irregular multifiber networks. We apply the MLLC model to the NSFnet shown in Fig. 8 and show the results in Fig. 9. The traffic load per node is 12 Erlangs. It is seen that the analytical results follow the trend of the simulation results. No result of the independence model is shown in the figure because the independence model is not applicable in irregular networks. We observed that the network performance is not significantly improved with full wavelength conversion ($F = 24$, $W = 1$) in the NSFnet compared to no wavelength conversion ($F = 1$, $W = 24$). It is also interesting to note that only 4 fibers per link are sufficient to provide similar performance as that of using full wavelength converters in the NSFnet.

From the above simulation and analytical results, we know that the MLLC model is applicable in both regular and irregular networks. The analytical results are close to the simulation results in all of the three examined topologies. We also observed that good

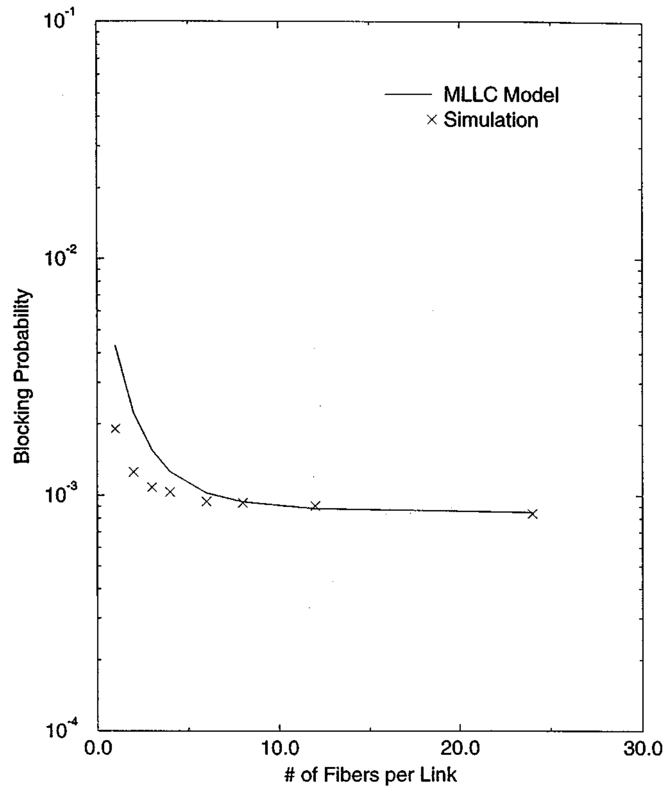

Fig. 9. Blocking probability versus number of fibers in the NSfnet. The link capacity is fixed at 24.

blocking performance is guaranteed in multifiber networks with a small number of fibers per link in different topologies.

IV. Conclusion

We study the effect of multiple fibers in circuit-switched all-optical WDM networks. To evaluate the blocking performance of such networks, we have developed an analytical model taking the link-load correlation into account. We have shown that the model is accurate for a variety of network topologies by comparing the analytical results to the simulation results. Comparing to the independence model of [16], the MLLC model is more accurate both in regular networks (the ring, the mesh-torus) and irregular networks (the NSFnet). An important conclusion of our study is that a multifiber network has similar blocking performance to that of a full-wavelength-convertible network, if we select the wavelength-fiber-pairs adequately. A limited number of fibers is sufficient to guarantee high network performance. Most of the current optical networks are built on multiple fibers. Multifiber WDM networks without wavelength conversion are not only feasible, but are also a desirable choice under current technologies.

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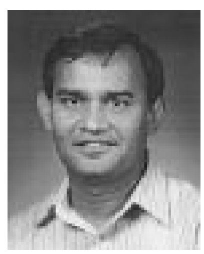

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